RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2025
THIRD YEAR [BATCH 2022-25]
PHYSICS (HONOURS)
Paper : CC 13

Date : 05/05/2025
Time : 11 am – 1 pm
Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Consider a gas consisting of 1000 atoms in an isolated container. Each atom can be in any one of the two possible energies 0 unit and 1 unit. The total energy of the system is 100 unit. Estimate the number of microstates by stating the importance of Stirling's approximation. You may keep the final answer in powers of *e*. (Hint: ln(10)=2.3, ln(3)=1.1)
   b) Show that the Shannon entropy (also known as uncertainty function) $H = -\sum_i p_i \, ln(p_i)$ is maximum when all microstates are equally likely. Here p_i is the probability that the system is in microstate i.
   c) A particle is moving in a one dimensional interval of length *L*. The probability that the particle is at a particular position in this interval is uniform. Find out its Shannon entropy. If the length of the interval is multiplied by a factor α, what will be the new entropy ? Interpret the results for $\alpha > 1$ and $\alpha < 1$. [4+2+4]

2. a) Consider two particles moving in an one dimensional interval of length *L*. Their only interaction is via exclusion i.e. both particles cannot occupy the same position at same time, otherwise they are non-interacting. The positions of the particles are x_1 and x_2, corresponding momenta are p_1 and p_2, respectively. Draw phase space diagrams in x_1-x_2 plane and p_1-p_2 plane.
   b) Derive the relation $Z_c(T,V,N) = \int dE \, e^{-\beta E} Z_{mc}(E,V,N)$, where Z_c and Z_{mc} are canonical and microcanonical partition functions of a system respectively, E is energy and $\beta = \frac{1}{kT}$ where *T* is the temperature of the system.
   c) Consider a three level system with corresponding energies $-\epsilon, 0, \epsilon$ at temperature *T*. Calculate the partition function, average energy, fluctuation of energy and specific heat of the system. [2+3+(1+1+2+1)]

3. a) Consider a gas of *N*-non-interacting molecules contained in a volume *V* at temperature *T*. Find the partition function for such system. Use partition function to find the energy of the system and find also the entropy of the system.
   b) Find the Helmholtz free energy (*F*) and hence show that it follows the ideal gas equation. [(2+2+2)+(2+2)]

4. Consider a collection of *N* non-interacting ideal gas particles with chemical potential μ in volume *V* at temperature *T*, obeying the Hamiltonian $H = \sum_i \frac{p_i^2}{2m}$. The canonical partition function of ideal gas is $Z_c(T,V,N) = \frac{V^N}{N! \lambda_{th}^{3N}}$ with thermal wavelength $\lambda_{th} = \frac{h}{\sqrt{2\pi mkT}}$.
   a) Find out its partition function and free energy in grand canonical ensemble.
   b) Show that the entropy of the system is $S(T,V,\mu) = \langle N \rangle k \left(\frac{5}{2} - \beta\mu\right)$, where $\langle N \rangle$ is the average number of particles and $\beta = \frac{1}{kT}$ with k being the Boltzmann constant. Argue how this result indicates the negativity of the chemical potential of ideal gas. [2+2+(5+1)]

5. a) Consider a system of paramagnetic molecules. Each molecule has angular momentum J and have possible values of $(2J+1)$ degenerate states. The system is placed in an external magnetic field B (along z direction) so that the degeneracy is lifted and the system has two levels. Find the partition function of the system and hence total energy U. (Take $J = 1/2$).

   b) Find the specific heat of this system. Plot the specific heat as function of temperature. Explain the Schottky anomaly. [(3+3)+(2+1+1)]

6. a) Consider a mixed ensemble of spin-1/2 systems consisting of fractional populations $w(S_x,-)=a$, $w(S_y,+)=b$ and $w(S_z,-)=1-a-b$. Find out the density matrix in basis of S_z.
Calculate the ensemble averages $[S_x]$, $[S_y]$ and $[S_z]$ using $[A] = Tr(\rho A)$. Under what condition $[S_z] = 0$?

   b) Show that the time evolution equation of the density operator $\rho(t)$ for an ensemble of quantum systems is $i\hbar\frac{\partial\rho}{\partial t} = [H,\rho]$, where H is the Hamiltonian of the system.

   c) What are the forms of the density matrix for pure ensemble and completely random ensemble in suitable bases? [(4+1)+3+(1+1)]

7. a) Consider a system of two non-interacting particles. The possible set of states for each particle is given by $s=1,2,\ldots n$. Calculate the total number of configurations for the three distinct cases when the particles obey (i) Maxwell-Boltzmann statistics, (ii) Bose-Einstein statistics, (iii) Fermi-Dirac statistics. In each case, find the ratio of the number of configurations for which both particles occupy same state, to the number of configurations where the particles are in different states. Compare and interpret the results of the three cases.

   b) Find the mean occupation $\tilde{n}_i$ of an ideal Fermi system with energy ε_i and chemical potential μ. Plot the function $f(\varepsilon_i) = \frac{\tilde{n}_i}{g_i}$, a function of temperature for at least two temperatures $T = 0\,K$ and $T \neq 0K$, where g_i is the degeneracy in i-th state. [6+(3+1)]

8. a) The average number of particles in an ideal Bose gas in volume V at temperature T and chemical potential μ, is given by $N(T,V,z) = \frac{V}{\lambda_{th}^3} g_{\frac{3}{2}}(z) + \frac{z}{1-z}$, where $g_n(z) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}}{z^{-1}e^x-1}dx = \sum_{k=1}^{\infty}\frac{z^k}{k^n}$ is the polylogarithm function, $z = e^{\beta\mu}$ and $\lambda_{th} = \frac{h}{\sqrt{2\pi mkT}}$. Discuss how (i) $z = 1$ gives rise to Bose-Einstein condensation while (ii) $z < 1$ does not give condensation. Find an expression of critical temperature in a conserved system of N bosons.

   b) Consider a random walker on a one dimensional discrete lattice. At each step, the walker can move to its right or left with equal probability $p=1/2$. The starting lattice site is designated by $m=0$. What is the probability that after $N=70$ steps the walker is at $m=35$? Explain. Does the answer change if a new feature is added so that the walker may not move at all in some steps (in this case $p<1/2$) ? Justify your answer. [(2+2+2)+(2+2)]

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